ON CALCULATING THE OPTIMAL DIMENSIONS AND PERFORMANCE PARAMETERS OF SPRAY NOZZLES

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UDC 66.011+66.069.83

A procedure is shown for calculating the optimal dimensions and performance parameters of spray nozzles, taking into account the motion of particles or liquid droplets due to inertia as well as the location of the device.

The differential equations of motion are solved for particles or liquid droplets in the active zone, with the effect of inertia during deceleration ($V_0 \rightarrow 0$) included in the hydraulic drag coefficient under the assumption that the particles are spherical in shape and that their diameter and their density do not significantly change during the motion.

Design equations for calculating the time t and the path l_i of the motion of particles due to inertia are derived and used for determining the optimal nozzle dimensions as well as the diameter-to-height ratio.

1. Along the nozzle diameter (x-axis):

for a Reynolds number in the range $\mathrm{Re} \leq 0.2$

$$t_x = \tau \ln \left| \frac{V_i \sin \varphi}{V_{ix}} \right|, \tag{1}$$

where $\tau = d^2(\rho_2 - \rho_1)/18\mu$,

$$l_{ix} = \tau (V_i \sin \varphi - V_{ix}); \tag{2}$$

for a Reynolds number in the range $\mathrm{Re} \leq 300$

$$t_x = \gamma \ln \left| \frac{V_i \sin \varphi (\alpha V_{ix} + \beta)}{V_{ix} (\alpha V_i \sin \varphi + \beta)} \right|, \tag{3}$$

where $\alpha = 0.75A\rho_1/d\rho_2$; $\beta = 0.75B\mu/d^2\rho_2$; $\gamma = 4d^2(\rho_2 - \rho_1)/3B\mu$;

$$l_{ix} = L \ln \left| \frac{\alpha V_i \sin \varphi + \beta}{\alpha V_{ix} + \beta} \right|, \tag{4}$$

where $L = 4d(\rho_2 - \rho_1)/3A\rho_1$.

2. Along the vertical nozzle axis (y-axis), with the transverse gas velocity V_G taken into account:

$$t_y = D^{-1} \ln \left| \frac{(C - D)(E + D)}{(C + D)(E - D)} \right|, \tag{5}$$

where $C = 2\alpha(V_i \cos \varphi \pm V_G) + \beta$; $D = (\beta^2 - 4\alpha g)^{0.5}$; $E = 2\alpha V_s + \beta$;

$$l_{iy} = \alpha^{-1} \ln F \mp (P \pm V_G) D^{-1} \ln \left| \frac{(C - D)(E + D)}{(C + D)(E - D)} \right|, \tag{6}$$

where $F = [\alpha(V_i \cos \varphi \pm V_G)^2 + \beta(V_i \cos \varphi \pm V_G) \pm g](\alpha V_s^2 + \beta V_s \pm g)^{-1}$; $P = B\nu/2Ad$.

The sedimentation rate of particles is calculated by the formula

$$V_s = 0.5 \left(-\alpha_1 + \sqrt{\alpha_1^2 + 4\beta_1}\right), \tag{7}$$

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where

$$\alpha_1 = B_s v/A_s d;\quad \beta_1 = 4dg(\rho_2 - \rho_1)/3A_s\rho_1.$$

It is shown that the initial velocity V_i of particles or liquid droplets, the spray jet angle 2φ and, particularly, the particle diameter 4 are decisive factors in the choice of nozzle dimensions and the diameter-to-height ratio. For particles with a diameter $d \geq 100\ \mu$ and $2\varphi = 60°$ the nozzle diameter is much larger than the height. In this case the particles remain suspended within the active nozzle zone for the rated length of time.

An example is shown to illustrate the calculation of optimal dimensions and performance parameters, also of the nozzle location and the spray intensity as functions of the dispersion spectrum of particles or liquid droplets.

NOTATION

A, B	are constants, respectively equal to 0.12 and 37 for the motion of particles or liquid droplets due to inertia with a Reynolds number Re $\leq$ 300;
A_s, B_s	are constants, respectively equal to 0.6 and 33 for droplets settling with a Reynolds number Re $\leq$ 300, and respectively equal to 0.402 and 40 for spherical solid particles;
d	is the diameter of particles or droplets, m;
ρ_1, ρ_2	are the densities of the medium and of the particles or droplets, respectively, kg/m^3;
η, ν	are the dynamic and kinematic viscosities of the medium, N·sec/m^2 and m^2/sec, respectively;
V_i, V_{ix}	are the initial and relative velocities of a particle, m/sec;
V_s	is the sedimentation rate of particles or droplets, m/sec;
x, y	are the coordinate axes along the nozzle diameter and the nozzle height, respectively, m;
g	is the gravitational constant, m/sec^2;
2φ	is the angle of the spray jet, ang. deg.

THERMAL STRESSES IN A HOLLOW CYLINDER OF FINITE LENGTH

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Along the inside surface (radius $r = r_0$) and the outside surface (radius $r = r_c$) of a hollow cylinder of finite length $0 \leq z \leq l$ and with transient internal heat sources of density $q(r, z, \tau)$ there flows a fluid whose temperature field $\varphi(z, \tau)$ is given. The heat transfer at the inside and the outside surface of this cylinder is effected by convection, but the respective heat transfer coefficients h_1 and h_2 have generally different values. The temperature of the end surfaces is the same as the temperature of the fluid.

The initial cylinder temperature $f(r, z)$ is a function of the space coordinates. The thermophysical properties are assumed constant. The problem is reduced to solving the equation

$$\frac{\partial\theta}{\partial\tau} = a\left\{\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) + \frac{\partial^2\theta}{\partial z^2}\right\} + \frac{\partial\varphi}{\partial\tau} - a\frac{\partial^2\varphi}{\partial z^2} - \frac{q}{c\rho} \tag{1}$$

with the following initial and boundary conditions:

$$\theta(r, z, 0) = \varphi(z, 0) - f(r, z),$$

$$\left.\frac{\partial\theta}{\partial r} - h_1\theta\right|_{r=r_0} = 0, \qquad \left.\frac{\partial\theta}{\partial r} + h_2\theta\right|_{r=r_c} = 0,$$

$$\theta|_{z=0} = 0, \qquad \theta|_{z=l} = 0,$$

where $\theta(r, z, \tau) = \varphi(z, \tau) - T(r, z, \tau)$.

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By successive finite Fourier-sine transformations and a finite Hankel transformation, the solution to this equation can be obtained in the form:

$$\theta(r,\ z,\ \tau)=\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{4y_n^2\,\overline{\theta}_{SH}\sin\frac{k\pi}{l}\,zV_0\left(y_n\frac{r}{r_c}\right)}{r_c^2\,l\left\{V_0^2(y_n)\left[y_n^2+Bi_2^2\right]-V_0^2(my_n)\left[m^2y_n^2+\mathrm{Bi}_1^2\right]\right\}}$$

$$\overline{\theta}_{SH}=\exp\left(-\frac{ay_n^2}{r_c^2}\tau-\frac{ak^2\pi^2}{l^2}\tau\right)\int\limits_0^{\tau}\left\{\frac{r_c^2}{y_n^2}\left[\mathrm{Bi}_2V_0(y_n)+\mathrm{Bi}_1V_0(my_n)\right]\right.$$

$$\times\int\limits_0^{l}\left(\frac{\partial\varphi}{\partial\eta}-a\frac{\partial^2\varphi}{\partial z^2}\right)\sin\frac{k\pi}{l}\,zdz-\frac{1}{c\rho}\int\limits_0^{l}\sin\frac{k\pi}{l}\,zdz\int\limits_{r_0}^{r_c}rq(r,\ z,\ \eta)$$

$$\left.\times V_0\left(y_n\frac{r}{r_c}\right)dr\right\}\exp\left(\frac{ay_n^2}{r_c^2}\eta+\frac{ak^2\pi^2}{l^2}\eta\right)d\eta+\left\{\frac{r_c^2}{y_n^2}\left[\mathrm{Bi}_2V_0(y_n)+\mathrm{Bi}_1V_0(my_n)\right]\right.$$

$$\left.\times\int\limits_0^{l}\varphi(z,\ 0)\sin\frac{k\pi}{l}\,zdz-\int\limits_0^{l}\sin\frac{k\pi}{l}\,zdz\int\limits_{r_0}^{r_c}rf(r,\ z)V_0\left(y_n\frac{r}{r_c}\right)dr\right\}\exp\left(-\frac{ay_n^2}{r_c^2}\tau-\frac{ak^2\pi^2}{l^2}\tau\right),$$

$$V_0\left(y_n\frac{r}{r_c}\right)=\left[y_nY_1(y_n)-\mathrm{Bi}_2Y_0(y_n)\right]J_0\left(y_n\frac{r}{r_c}\right)-\left[y_nJ_1(y_n)-\mathrm{Bi}_2J_0(y_n)\right]Y_0\left(y_n\frac{r}{r_c}\right),$$

where y_n are the roots of the transcendental equation

$$my V_1(my)=-\mathrm{Bi}_1V_0(my). \tag{2}$$

The first six roots of Eq. (2) are calculated here for the case where both referred heat transfer coefficients are equal ($h_1 = h_2$) with $m = r_0/r_c = 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9$ and $\mathrm{Bi}_2 = 2,\ 5,\ 10,\ 15,\ 20,\ 30,\ 40,\ 50,\ 60,\ 80,\ 100,\ 200,\ 300$.

The corresponding thermoelasticity problem is solved by the Goodyear method.

NOTATION

$T(r,\ z,\ \tau)$	is the cylinder temperature;
a	is the thermal diffusivity;
c	is the specific heat of the material;
ρ	is the density of the material;
$\mathrm{Bi}_1 = h_1/r_0$	is the Biot number for the inside surface;
$\mathrm{Bi}_2 = h_2/r_c$	is the Biot number for the outside surface.

SOLIDIFICATION OF A MELT IN A WATER-COOLED ANNULAR CAVITY

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UDC 536.21

An approximate solution is given to the problem of bilateral solidification in an annular cavity and is applied to the conditions of continuous casting of hollow ingots. In practice this process follows the conventional scheme [1]: molten metal is poured from the top down into the gap between cylindrical walls of an inner and an outer crystallizer, whereupon the frozen ingot is continuously pulled down at a constant speed. Below the crystallizers it passes through a secondary cooling zone and there its solidification is completed.

N. I. Lobachevskii State University, Gor'kii. Original article submitted February 15, 1971; abstract submitted September 30, 1971.

Mathematically the problem is solved by the well-known Leibenzon approximation method [2]. Within this approximation one determines the coordinate of approach (ξ_0) between the outer and the inner crystallization front:

$$\varepsilon_0^2=\left(\frac{\xi_0}{R_1}\right)^2=\frac{k^2-1}{2\ln k}+\frac{1-\varepsilon_0^2+\varepsilon_0^2\ln\varepsilon_0^2}{2\ln k}\left(\frac{T_{s2}-T_{s1}}{T_f-T_{s1}}\right), \tag{1}$$

with the radius of the inner crystallizer R_1, the radius of the outer crystallizer R_2, $k = R_2/R_1$, the temperature at the ingot surface on the side of the outer crystallizer T_{s2}, and the temperature at the ingot surface on the side of the inner crystallizer T_{s1}.

It is indicated that under practical conditions [1] $|T_{s2}-T_{s1}| \ll T_f-T_{s1}$ and a simpler version of formula (1) can be used

$$\varepsilon_0^2=\frac{k^2-1}{2\ln k}, \tag{2}$$

which involves only the ingot geometry.

The quantity ε_0 is one of the process parameters involved in the continuous casting of hollow ingots. With this parameter known, one determines the time required for complete solidification of a hollow ingot. Thus, in the approximation (2) one finds here that

$$\mathrm{Fo}=\frac{1}{4}\left\{1-\frac{k^2-1}{\ln k^2}+\frac{k^2-1}{\ln k^2}\ln\left(\frac{k^2-1}{\ln k^2}\right)\right\}\frac{\frac{\sigma}{c}+T_0-T_f}{T_f-T_{s1}}, \tag{3}$$

with the heat of crystallization σ, the specific heat c, the freezing temperature T_f, the temperature of the melt T_0, and the Fourier number Fo.

For illustration, we determine the total solidification time under the following conditions: $k = 10$, $T_0 = 1783°K$, $T_f = 1743°K$, and $T_{s1} = 1273°K$. Expression (3) yields $\mathrm{Fo} = 14.5$.

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A METHOD OF CALCULATING THE HEAT TRANSFER IN A PERIODIC-DUTY FURNACE

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UDC 669.046.4

A method is proposed by which the external and the internal heat transfer can be calculated at the same time for a periodic-duty furnace operating in identical cycles: cold metal is placed inside the furnace and taken out after it has been heated up to the desired temperature; this cycle is then repeated.

The temperature of the flame is assumed uniform over the volume; the temperature of the metal and the temperature of the furnace wall are assumed uniform over the surface. All furnace components are assumed to behave as gray bodies. On this basis, the external heat transfer is calculated by the zonal method with the heat balance in the chamber taken into account.

For calculating the internal heat transfer, the metal and the furnace wall are treated as infinitely large plates of finite thickness. The transient temperature field here, with boundary conditions of the

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second kind, is calculated with the aid of the finite Fourier-cosine transformation. Moreover, the thermal fluxes at the boundaries are determined from the external heat transfer. The calculation procedure is specially modified for the case of a "thin" metal piece.

The proposed method is useful for calculating the heat transfer in a furnace which operates under a constant heat load and at a constant wall temperature. A specific example is shown for illustration.

EFFECT OF A THIN HETEROGENEOUS INCLUSION ON A TWO-DIMENSIONAL STEADY-STATE FIELD

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UDC 536.2

The author analyzes the perturbation of a two-dimensional steady-state temperature field by a thin heterogeneous inclusion (or crack) whose thermal conductivity k_1 is different from the thermal conductivity k of the base medium.

The following conditions are derived from the law of heat conservation along an inclusion:

$$h_1 \frac{\partial \psi}{\partial s} = -\varepsilon (T^+ - T^-), \qquad \varepsilon = \frac{ak_1}{2kh_0}, \qquad k_1 < k, \tag{1}$$

$$h_1 \frac{\partial T}{\partial s} = \varepsilon_1 (\psi^+ - \psi^-), \qquad \varepsilon_1 = \frac{ak}{2k_1h_0}, \qquad k_1 > k. \tag{2}$$

with T^+, T^- and ψ^+, ψ^- denoting the limits of temperature T and of the flow function ψ as inclusion Γ is approached from the left side and from the right side, respectively, this inclusion having the length $2a$ and the width $2h_0h_1(s)$ at section s ($|s| < 1$, $h_0 = \mathrm{const}$).

The complex potential $W(z) = T + i\psi$, which will satisfy condition (1) along Γ and which will converge to a given analytic function F(z) at a far distance from Γ is sought for $\varepsilon \ll 1$ in the form of a series in integral positive powers of parameter ε:

$$W(z) = F(z) + \sum_{k=0}^{\infty} \varepsilon^k W_k(z), \qquad W_k(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{\omega_k(t)\,dt}{t - z}. \tag{3}$$

For determining the unknown densities $\omega_k(t)$ one obtains here a system of singular integrodifferential equations.

In greater detail is considered the case where Γ is a straight-line segment and it is proved that series (3) converges uniformly when $\varepsilon < \pi/2N$,

$$N < 7L, \qquad L = \max_{|s|<1} [\sqrt{1-s^2}/h_1(s)]. \tag{4}$$

When $\varepsilon \gg 1$, the complex potential is sought in the form of a series in integral negative powers of ε.

To illustrate this method of analysis, one considers here the effect of a poorly conducting flat elliptical inclusion on a uniform temperature field, on a thermal flux bending around a right angle, and on a source-type field. For a well conducting inclusion, i.e., for boundary condition (2), the problem is solved analogously.

It is pointed out that similar problems arise in the filtration theory, in electrostatics, in magnetostatics, and in the analysis of steady-state electric fields produced by currents.

Institute of Physics, Dagestan Branch, Academy of Sciences of the USSR, Makhachkala. Original article submitted February 9, 1971; abstract submitted August 24, 1971.

STEADY-STATE TEMPERATURE DISTRIBUTION ON A WEDGE SURFACE WITH MIXED-TYPE BOUNDARY CONDITIONS

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UDC 536.21

An effective method of solving problems in potential theory with mixed-type boundary conditions is the method of paired integral equations with a subsequent reduction to a one-dimensional integral Fredholm equation of the second kind with a symmetric kernel [1-4].

The steady-state surface temperature distribution is considered in the case of a wedge whose one plane remains at a constant temperature while the other plane is thermally insulated over a finite region adjoining the common edge but emits heat according to Newton's law from the rest of its area into a medium at zero temperature.

In cylindrical coordinates this problem reduces to solving the Laplace equation for the following boundary conditions:

$$\begin{aligned} &T = T_0, \quad \varphi = 0, \quad 0 \leqslant \rho < \infty, \\ &\frac{1}{\rho} \cdot \frac{\partial T}{\partial \varphi} = 0, \quad \varphi = \gamma, \quad 0 < \rho < a, \\ &\frac{1}{\rho} \cdot \frac{\partial T}{\partial \varphi} + hT = 0, \quad \varphi = \gamma, \quad a < \rho < \infty. \end{aligned} \tag{1}$$

Here h is a positive constant. With the aid of functions constructed earlier in [5, 6], the problem is reduced further to the integral equation

$$T(\rho) = T_1(\rho) + h \int_0^a T(r) K_\gamma^{(-)}(r\rho)\, dr, \quad 0 \leqslant \rho \leqslant a. \tag{2}$$

Here $T(\rho)$ is the unknown temperature distribution on the wedge surface $\varphi = \gamma$. For wedge angles $\gamma = \pi/2m$ ($m = 1, 2, 3\ldots$) function $T_1(\rho)$ and kernel $K_\gamma^{(-)}(r\rho)$ are expressed in closed form in terms of an integral exponential function, and an effective numerical solution of Eq. (2) becomes possible.

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F. Engels Institute of Soviet Commerce, Leningrad. Original article submitted April 7, 1971; abstract submitted September 5, 1971.

BASING THE FUNDAMENTAL LAWS OF FILTRATION ON THE EXTREMALITY PRINCIPLES IN THE THERMODYNAMICS OF IRREVERSIBLE PROCESSES

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When a viscous filler fluid flows through pores in an industrial process bed, then a macro-unbalanced process will occur due to the presence of irreversible thermodynamic forces and fluxes.

In order to establish a relation between the irreversible thermodynamic forces and fluxes in a physically nonlinear situation (filtration in the case where Darcy's linear law is not valid), one must apply the principle of minimal irreversible forces (G. Ziegler), according to which the real velocity $\dot{x}_k$ yields the minimum irreversible forces $X_k^{(i)}$ which satisfies the condition

$$X_k^{(i)} \dot{x}_k = D(\dot{x}_k) \geqslant 0.$$

when the magnitude of the dissipation function and the direction of the irreversible force are known. On the basis of this principle, it is possible to establish a relation between irreversible forces and velocities in the form

$$X_k^{(i)} = \eta \frac{\partial D}{\partial \dot{x}_k}. \tag{1}$$

Here $\eta = ((\partial D/\partial \dot{x}_j)\dot{x}_j)^{-1} D$ with D denoting the dissipation function. Assuming that an irreversible force $X^{(i)} = -\operatorname{grad} \varphi$ (φ denoting the filtration potential) and introducing a dissipation function in the form

$$D = \frac{1}{2} \sum_{i=1}^{N} \alpha_i v_i^2 + \frac{1}{3} \sum_{i=1}^{N} \beta_i |v_i|^3, \tag{2}$$

one obtains (for a one-dimensional flow in the bed) a nonlinear relation expressed as

$$-\frac{d\varphi}{dx} = \frac{\alpha}{2} \dot{x} + \frac{\beta}{3} (\dot{x})^2 \operatorname{sgn} \dot{x}. \tag{3}$$

For a three-dimensional flow in an anisotropic bed, based on a dissipation function

$$D = \alpha_{ij} \dot{x}^i \dot{x}^j + |\beta_{ijk} \dot{x}^l \dot{x}^j \dot{x}^k|, \tag{4}$$

we have

$$-\frac{\partial \varphi}{\partial x^i} = \alpha_{ij} \dot{x}^j + \beta_{ijk} \dot{x}^j \dot{x}^k \operatorname{sgn}(\dot{x}^i). \tag{5}$$

Here α_{ij} and β_{ijk} are tensors of the filtration resistance corresponding to linear-law and to nonlinear-(square)-law filtration. By defining the dissipation function in still more general terms, one obtains the universal relation in the form

$$-v^k \frac{\partial \varphi}{\partial x^k} = \exp(\gamma_j v^j) - 1.$$

Groznyy Petroleum Institute, Groznyy. Original article submitted September 10, 1969; abstract submitted September 14, 1971.

UNIAXIAL ELONGATION OF ULTRAELASTIC POLYVINYL CHLORIDE

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UDC 678.01:532.135

Relations between real stress and elongation have been established for polyvinyl chloride fibers at temperatures in the 90-160°C range. Deformations were produced in two modes: at constant rates of elongation (v = 0.0079, 0.078, 0.769, and 8.33 mm/sec) and at a constant rate of strain ($\dot{\varepsilon} = 0.002\ \mathrm{sec}^{-1}$).

Based on the results of elastic recovery tests, the total elongation of a specimen (α) was split into an ultraelastic component (α_e) and an irreversible component (α_f). If the initial length of a specimen is l_0, its length after deformation is l, and its length after elastic recovery is l', then we have $\alpha = l/l_0 = (l/l')(l'/l_0) = \alpha_e\alpha_f$.

The real stress and the ultraelastic deformation are both extremal functions of α. The occurrence of maxima here is explained by a stress relaxation and by processes which result in a gradual breakdown of a specimen.

Ultraelastic deformation was found to be the principal component of total deformation in the case of polyvinyl chloride tested for elongation in those two modes.

The dependence of stress on the magnitude of ultraelastic deformation at temperatures far from the vitrification point is accurately enough described by the equation $P = E(\alpha_e^2 - \alpha_e^{-1})$ (Fig. 1).

It has been established that the ultimate strength of polyvinyl chloride fibers rapidly vitrified after a certain definite elongation will depend on the amount of accumulated ultraelastic deformation.

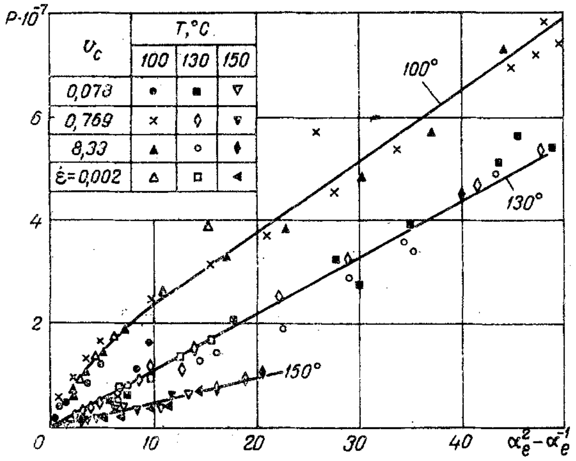

Fig. 1. Real stress as a function of ultraelastic deformation.

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ONE PROBLEM OF HEAT CONDUCTION WITH MIXED-TYPE BOUNDARY CONDITIONS AND A POINT SOURCE OF HEAT

M. G. Strunskii

UDC 536.2.01

An infinite plane is considered on which the surface of a circular disk with the radius a is thermally insulated while convective heat transfer occurs from all the remaining surface to the surrounding medium; on the disk axis at a distance z_0 below the plane is located a point source emitting a thermal flux Q.

In order to determine the steady-state temperature field in the homogeneous medium containing the point source of heat, it is necessary to integrate the Poisson equation

$$\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial U}{\partial r}+\frac{\partial^2 U}{\partial z^2}=-\frac{\operatorname{div} q}{\lambda},$$

with $q = 0$ everywhere except at point $(z = z_0,\ r = 0)$ at which $q = \infty$ and with the boundary conditions

$$\frac{\partial U}{\partial z}=0 \text{ for } z=0,\ r<a,$$

$$U-h\frac{\partial U}{\partial z}=0 \text{ for } z=0,\ r>a,$$

where

$$h=\frac{\lambda}{\alpha}=\text{const}>0.$$

The solution to this problem, obtained by the method of integral transformations and on the basis of a few special theorems, has the form

$$U=\frac{Q}{4\pi\lambda}\left[\frac{1}{\sqrt{r^2+(z-z_0)^2}}+\frac{1}{\sqrt{r^2+(z+z_0)^2}}\right]+\int_0^\infty\left[A(p)-\frac{Q}{2\pi\lambda}\exp(-z_0p)\right]\frac{\exp(-zp)\,J_0(rp)}{1+hp}\,dp,$$

with the unknown coefficient $A(p)$ found by the formula

$$A(p)=\int_0^a \varphi(t)\cdot\sin pt\,dt.$$

The auxiliary function $\varphi(t)$ is determined from the Fredholm equation of the second kind

$$\varphi(t)+\frac{2}{\pi}\int_0^a K(t,\tau)\,\varphi(\tau)\,d\tau=f(t)$$

with the continuous kernel

$$K(t,\tau)=\frac{1}{2h}\left[\sin\frac{t+\tau}{h}\operatorname{si}\frac{t+\tau}{h}+\cos\frac{t+\tau}{h}\operatorname{Ci}\frac{t+\tau}{h}-\sin\frac{|t-\tau|}{h}\operatorname{si}\frac{|t-\tau|}{h}-\cos\frac{|t-\tau|}{h}\operatorname{Ci}\frac{|t-\tau|}{h}\right]$$

and the free term

$$f(t)=\frac{Q}{\pi^2\lambda}\int_0^\infty\frac{\exp(-z_0p)\sin pt}{1+hp}\,dp.$$

Various special and limiting cases of the problem are considered.

When $z_0 = z = 0$, for instance, i.e., when the heat source lies at the center of the thermally insulated circle and when only the temperature field in the boundary plane is of concern, the temperature is determined according to the equation

$$U|_{z_0=z=0}=\frac{Q}{2\pi\lambda}\left\{\frac{1}{r}-\frac{\pi}{2}\cdot\frac{1}{h}\left[H_0\left(\frac{r}{h}\right)-N_0\left(\frac{r}{h}\right)\right]\right\}+\int_0^\infty A(p)|_{z_0=0}\frac{J_0(rp)}{1+hp}\,dp.$$

Coefficient $A(p)|_{z_0=0}$ is found here by the same formulas as in the general case, except the formula for the free term now expressed in explicit form.

It is easy to see that the effect of a thermally insulated region in this case is characterized by the integral term alone; when $a = 0$, i.e., without thermal insulation, coefficient $A(p)|_{z_0=0} = 0$ and the temperature distribution is determined by the term inside the large bracket alone.

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It is pointed out, in conclusion, that the problem analyzed here arises also in other applications of potential theory as, for example, in calculating the steady-state electric field produced by linearly polarizing electrodes.

NOTATION

U	is the temperature;
Q	is the thermal flux;
q	is the thermal flux density;
h	is the material parameter with the dimension of length;
λ	is the thermal conductivity;
α	is the heat transfer coefficient;
a	is the radius of thermal insulation;
z_0	is the distance from the point source of heat to the boundary plane;
r, z	are the cylindrical coordinates;
f, φ	are the function signs;
si, Ci	are the integral-sine and integral-cosine, respectively;
J_0	is the zeroth-order Bessel function of the first kind;
H_0, N_0	are the Struve function and the Neumann function, respectively.

DETERMINING THE HEAT TRANSFER COEFFICIENT IN TRANSIENT HEAT CONDUCTION PROBLEMS ON THE BASIS OF MEASURED SURFACE TEMPERATURE

O. T. Il'chenko and L. I. Shifan

UDC 536.21

The problem considered here is that of determining the parameters of heat transfer at a surface from temperature measurements.

In the case of a symmetrical problem, the solution is sought in the form [1]

$$U(\bar{x}, t) = \int_0^t \frac{\mu(\tau)}{\sqrt{\pi(t-\tau)}} \left[\exp\left\{ -\frac{(\bar{x}+1)^2}{4\bar{a}^2(t-\tau)} \right\} + \exp\left\{ -\frac{(\bar{x}-1)^2}{4\bar{a}^2(\ -\tau)} \right\} \right] d\tau. \tag{1}$$

Applying in succession the Abel and the Laplace transformation, we obtain an integral equation for the thermal potential function $\mu(t)$ in terms of function $U(\bar{x}, t)$ values.

Inserting $\mu(t)$ into the boundary condition, with the aid of the Abel transformation, we obtain an integral equation in the unknown function Bi(t):

$$\int_0^t \frac{\mathrm{Bi}(\tau)\,[U_c(\tau) - U(1, \tau)]}{\sqrt{t-\tau}}\, d\tau = \frac{\sqrt{\pi}}{a} U(1, t) + \frac{2}{a^2} \sum_{n=1}^{\infty} (-1)^n \int_0^t \frac{U(1, \tau)\, n}{\sqrt{(t-\tau)^3}} \exp\left\{ -\frac{n^2}{a^2(t-\tau)} \right\} d\tau. \tag{2}$$

Unlike certain analytic solutions [2, 3], this equation solved for Bi(t) does not contain derivatives of temperature functions at the surface. This certainly improves the accuracy of the solution, since derivatives of a function established experimentally cannot always be calculated within sufficient accuracy.

The procedure for solving the integral equation (2) is shown with examples applicable to the solution of inverse problems.

Khar'kov Polytechnic Institute, Khar'kov. Original article submitted May 20, 1971; abstract submitted October 19, 1971.

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A HEAT-CONDUCTING ROD IN A MEDIUM WITH AN ALTERNATING TEMPERATURE

I. M. Shnaid

UDC 536.2

The linear heat conduction problem is analyzed for the case of a thin long rod in a medium (I) whose temperature t_1 is a periodic function of time τ. One end of the rod is free; the other end is in thermal contact with a heat conducting wall, where heat is transferred directly from medium I and to medium II having a constant temperature t_2. On the basis of these stipulations, the rod temperature is described by the differential equation

$$\frac{c\rho f}{\alpha p}\cdot\frac{\partial t}{\partial\tau}+\frac{\lambda f}{\alpha p}\cdot\frac{\partial^2 t}{\partial x^2}+t=t_1(\tau) \qquad (1)$$

with the uniqueness conditions

$$\left.\frac{\partial t}{\partial x}\right|_{x=0}=0, \qquad (2)$$

$$\left.\left[\frac{\partial t}{\partial\tau}+\frac{\lambda f}{C_w}\cdot\frac{\partial t}{\partial x}+\frac{1}{C_w}\left(\frac{1}{R_{w1}}+\frac{1}{R_{w_2}}\right)t\right]\right|_{x=l}=\frac{t_1(\tau)}{R_{w_1}C_w}. \qquad (3)$$

In order to obtain a solution which is periodic in time, one expresses functions $t_1(\tau)$ and $t(x, \tau)$ in the form

$$t_1(\tau)=\sum_{n=0}^{\infty}T_{1n}\exp(i\omega_n\tau),$$

$$t(x,\tau)=\sum_{n=0}^{\infty}T_n(x)\exp(i\omega_n\tau),$$

where $i=\sqrt{-1}$ and $\omega_n = 2\pi n/\tau_0$.

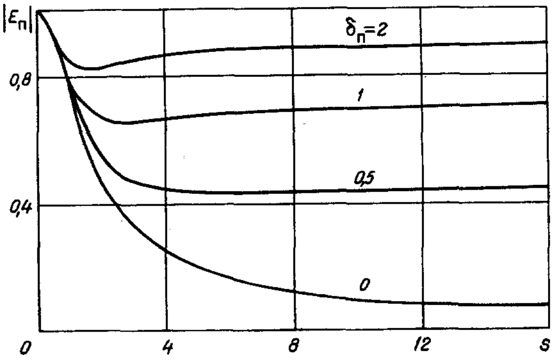

Fig. 1

Odessa Engineering Institute of the Refrigeration Industry. Original article submitted September 3, 1970; abstract submitted October 19, 1971.

From Eq. (1) and the boundary conditions (2), (3) one determines the coefficients $T_n(x)$ and then the instantaneous values of thermal power Q_p, Q_f transmitted through the lateral surface and the end surface of the rod, respectively:

$$Q_p = \int_0^l \alpha p\,(t_1 - t)\,dx = \sum_{n=0}^{\infty} Q_{pn} \exp(i\omega_n\tau),$$

$$Q_f = -\lambda f \left.\frac{\partial t}{\partial x}\right|_{x=l} = \sum_{n=0}^{\infty} Q_{fn} \exp(i\omega_n\tau).$$

The most characteristic example is analyzed here, namely where the fluctuations of the wall temperature are negligible. Then

$$T_n = \frac{T_{1n}}{1+i\delta_n}\left[1 - \frac{\operatorname{ch}(a_n+b_n i)\,sz}{\operatorname{ch}(a_n+b_n i)\,s}\right], \tag{4}$$

$$Q_{pn} = \alpha p l\,\frac{T_{1n}}{1+i\delta_n}\left[i\delta_n + \frac{\operatorname{th}(a_n+b_n i)\,s}{(a_n+b_n i)\,s}\right], \tag{5}$$

$$Q_{fn} = \alpha p l\,\frac{T_{1n}}{(a_n+b_n i)\,s}\cdot\frac{\operatorname{sh}(a_n+b_n i)\,sz}{\operatorname{ch}(a_n+b_n i)\,s}. \tag{6}$$

with

$$a_n = \frac{1}{\sqrt{2}}\left[\left(1+\delta_n^2\right)^{\frac{1}{2}} + 1\right]^{\frac{1}{2}};\quad b_n = \frac{1}{\sqrt{2}}\left[\left(1+\delta_n^2\right)^{\frac{1}{2}} - 1\right]^{\frac{1}{2}};$$

$$\delta_n = \frac{\omega_n c\rho f}{\alpha p};\quad s = \left(\frac{\alpha p}{\lambda f}\right)^{\frac{1}{2}} l;\quad z = \frac{x}{l}.$$

The relations derived here indicate that the performance of a heat regenerator with a regular wire (rod) mesh depends appreciably on the thermal conductivity of the latter and on the heat transfer at the walls. Only at high values of parameter s does the effect of thermal conductivity and heat transfer at the walls become negligible.

When the temperature of a medium varies periodically, then the effectiveness of a fin $|E_n| = |Q_{pn}/Q_{pn}|_{s=0}|$ must be determined for each harmonic separately as a function of s and δ_n. At s = const, according to Fig. 1, a large δ_n increases the effectiveness of a fin. It is interesting to note that the curves of $|E_n|$ vs s pass through minima. The value $s = s_e$ which corresponds to $|E_n|_{\min}$ increases infinitely as $\delta_n \to 0$. When $s > s_e$ and $\delta_n \neq 0$, a longer fin is more effective, while at the same time

$$\lim_{s\to 0} |E_n| = \frac{\delta_n}{\left(1+\delta_n^2\right)^{\frac{1}{2}}}.$$

NOTATION

x	is the rod length coordinate;
l, f, p	are the length, cross section area, and perimeter of a rod, respectively;
λ, ρ, c	are the thermal conductivity, density, and specific heat of rod material, respectively;
α	is the heat transfer coefficient;
C_W	is the total heat capacity of a wall;
R_{W1}, R_{W2}	are the thermal resistance from the wall to medium I and from the wall to medium II, respectively;
τ_0	is the period of function $t_1(\tau)$;
n	is the number of the harmonic.

TEMPERATURE FIELD AND ELECTRIC FIELD IN A PLANE LAYER WITH A TEMPERATURE-DEPENDENT RESISTANCE

R. S. Kuznetskii

UDC 538.3+536.212.2

The steady-state temperature field t(x) and electric field e(x) in a conductor, in the one-dimensional analysis disregarding any thermoelectric effects, are described by the following system of equations:

$$\lambda \rho t'' = -e^2, \quad e/\rho = j = \text{const}, \tag{1}$$

according to which the field is proportional to ρ; the thermal conductivity is considered constant and the electrical resistivity a function of the temperature $\rho = \rho(t)$.

An integration of the nonlinear equations (1) yields an expression which can be written as

$$|x - x_m| = \frac{1}{j} \sqrt{\frac{\lambda}{2}} \int_t^{t_m} \frac{dt}{\sqrt{\int_t^{t_m} \rho dt}}, \quad e = j\rho\,[t(x)), \tag{2}$$

with the coordinate x_m corresponding to the only temperature extremum: a maximum (t_m). Both parameters are determined from the boundary conditions.

When $\rho = \rho(1 + \alpha t)$, for instance, we have closed expressions for t and e as well as for the thermal flux density and the electric potential:

$$t = \frac{1}{\alpha}\left[(1+\alpha t_m) \cos j\,(x - x_m) \sqrt{\frac{\rho_0}{\lambda}\,\alpha} - 1\right], \quad e = \rho_0 j\,(1+\alpha t_m) \cos j\,(x - x_m) \sqrt{\frac{\rho_0}{\lambda}\,\alpha}; \tag{3}$$

$$q = j\,(1+\alpha t_m) \sqrt{\frac{\lambda\rho_0}{\alpha}} \sin j\,(x - x_m) \sqrt{\frac{\rho_0}{\lambda}\alpha}, \quad v = \int_x^{x_m} e\,dx = -\frac{1}{j}\,q. \tag{4}$$

We consider a conducting plane layer (a plate) or a prismatic (cylindrical) beam with a segment of the lateral surface $0 \le x \le a$ thermally insulated, with the temperatures $t(0) = 0$ and $t(0) = t_1 \ge 0$ and the electric current density j given. Moreover, x_m and t_m are determined from the equations

$$\int_0^{t_m} \frac{dt}{\sqrt{\int_0^{t_m} \rho dt}} = j|x_m| \sqrt{\frac{2}{\lambda}}, \quad \int_{t_1}^{t_m} \frac{dt}{\sqrt{\int_{t_1}^{t_m} \rho dt}} = j\,|a - x_m| \sqrt{\frac{2}{\lambda}} \tag{5}$$

or, when the electrical resistivity is a linear function of the temperature, from the equations

$$x_m = \frac{a}{\sqrt{\theta\alpha}} \operatorname{arctg} \frac{1 + \alpha t_1 - \cos\sqrt{\theta\alpha}}{\sin\sqrt{\theta\alpha}}, \quad t_m = \frac{1}{\alpha}\left[\frac{\sqrt{(\alpha t_1)^2 + 4\sin^2 \frac{1}{2}\sqrt{\theta\alpha}}}{\sin\sqrt{\theta\alpha}} - 1\right], \tag{6}$$

with $\theta = (aj)^2 \rho_0/\lambda$. For convenience, with the aid of identities

$$(1+\alpha t_m) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} \frac{x - x_m}{a} \sqrt{\theta\alpha} = \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} \frac{x}{a} \sqrt{\theta\alpha} \pm \frac{1 + \alpha t_1 - \cos\sqrt{\theta\alpha}}{\sin\sqrt{\theta\alpha}} \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} \frac{x}{a} \sqrt{\theta\alpha} \tag{7}$$

we transform expressions (3) and (4) so that they will not contain x_m and t_m.

For instance, in the case of boundary conditions symmetrical with respect to the center section ($t_1 = 0$) we have

$$x_m = \frac{a}{2}, \quad t_m = \frac{1}{\alpha}(\sec\sqrt{\theta\alpha} - 1), \quad \frac{1 + \alpha t_1 - \cos\sqrt{\theta\alpha}}{\sin\sqrt{\theta\alpha}} = \operatorname{tg} \frac{\sqrt{\theta\alpha}}{2}. \tag{8}$$

In conclusion, respective approximations of formulas (6), (3), and (4) are derived for the limiting case $\theta\alpha \ll 1$.

VNIPIChermeténergoochistka, Khar'kov. Original article submitted December 3, 1969; abstract submitted October 28, 1971.

MASS TRANSFER DURING CHEMICAL INTERACTION BETWEEN A SOLID AND A LIQUID ACCOMPANIED BY EMISSION OF GAS

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UDC 66.015.23

A theoretical and experimental study has been made concerning the kinetics of heterogeneous chemical processes which occur in a solid–liquid system accompanied by gas emission in the form of bubbles.

It has been shown in [1] that two kinds of effects result for bubbling: 1) the emitted gas bubbles displace some liquid and, as a consequence, the rate of diffusion-limited heterogeneous reactions increases; 2) the generated gas bubbles shield part of the solid surface and, as a consequence, the total diffusion is reduced.

Owing to a combination of both effects, an increasing mass flow density causes the mass transfer coefficient first to increase and then to decrease.

Assuming that the kinetic laws of mass transfer under the given conditions of a heterogeneous reaction are identical to the laws of heat transfer during bubble boiling of a liquid, the authors define a system of parameters respectively paired for the two analogous processes:

heat exchange:	mass exchange:	
$l \sim \sqrt{\frac{\sigma}{g(\rho' - \rho'')}}$;	$l \sim \sqrt{\frac{\sigma}{g(\rho' - \rho_G)}}$,	(1)
$q = \alpha \Delta t$;	$q_G = k_G \Delta c_G$;	(2)
$w = \frac{q}{r\rho''}$;	$w = \frac{q_G}{\rho_G}$.	(3)

Equation (3) follows from the relation

$$-D_G\left(\frac{\partial c}{\partial R}\right)_S = \rho_G \frac{dR}{d\tau}. \tag{4}$$

The excess of gaseous products Δc_G near the reaction surface is established on the basis of the reaction balance

$$nk_G(c_G - c_{Gs}) = k_R (c_R - 0). \tag{5}$$

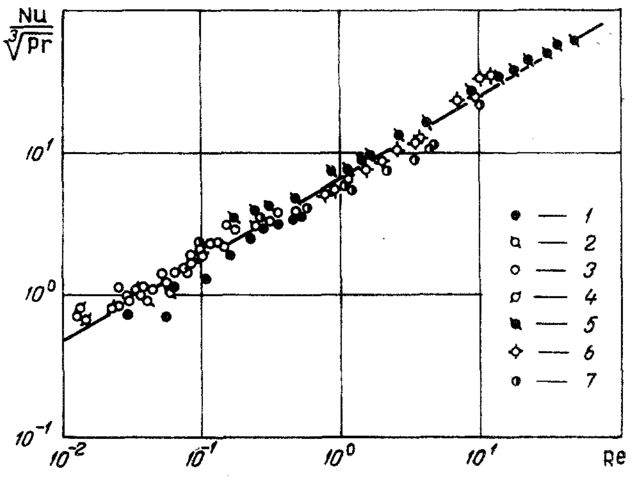

Fig. 1. Generalization of test data according to Eq. (9) on the interaction between $CaCO_3$, $(CuOH)_2CO_3$, Mg, Mn, and acids: 1) $CaCO_3 + HCl$; 2) $CaCO_3 + HNO_3$; 3) $(CuOH)_2CO_3 + HCl$; 4) $(CuOH)_2CO_3 + HNO_3$; 5) $Mg + HCl$; 6) $Mg + H_2SO_4$; 7) $Mn + HCl$.

L'vov Polytechnic Institute, L'vov. Original article submitted September 20, 1971; abstract submitted October 29, 1971.

Considering that [2]

$$\frac{k_R}{k_G}=\frac{D_R}{D_G}\sqrt[3]{\frac{\mathrm{Pr}_R}{\mathrm{Pr}_G}}=\left(\frac{D_R}{D_G}\right)^{2/3}, \tag{6}$$

we have for the concentration drop of dissolved gas

$$\Delta c_G=c_G-c_{Gs}=\frac{c_R}{n}\left(\frac{D_R}{D_G}\right)^{2/3}. \tag{7}$$

It appears feasible, by extending this analogy, to seek the mass transfer coefficient in the form

$$\mathrm{Nu}=f(\mathrm{Re},\ \mathrm{Pr}), \tag{8}$$

where

$$\mathrm{Nu}=\frac{k_G l}{D_G}=\frac{q_G n l}{c_R D_R}\sqrt[3]{\frac{D_R}{D_G}};\ \mathrm{Pr}=\frac{\nu}{D_G};\ \ \mathrm{Re}=\frac{wl}{\nu}=\frac{q_G l}{\rho_G \nu}$$

The kinetics of mass transfer during bubbling was studied in typical interactions between calcite, malachite, magnesium, manganese and sulfuric acid, hydrochloric acid, nitric acid in various concentrations.

The test data were evaluated and plotted on a curve fitting the criterial relation

$$\mathrm{Nu}=6.9\mathrm{Re}^{0.57}\mathrm{Pr}^{1/3} \tag{9}$$

This equation does qualitatively resemble the analogous equation which Kutateladze [3] and Labuntsov [4] have obtained for heat transfer. This justifies the use of such heterogeneous reactions as "cold" models in simulating heat transfer processes (during boiling).

NOTATION

c_G	is the concentration of gas at the reaction surface, kg/m^3;
c_{Gs}	is the saturation concentration of gas in a volume of liquid, kg/m^3;
c_R	is the concentration of hydrogen ions, kg/m^3;
D_G	is the diffusivity of gas, m^2/sec;
D_R	is the diffusivity of hydrogen ions, m^2/sec;
q	is the thermal flux density, W/m^2;
q_G	is the mass flow density of gas, kg/m^2;
w	is the characteristic linear velocity;
l	is the characteristic linear dimension, proportional to the breakaway diameter of a bubble, m;
k_G, k_R	are the mass transfer coefficients, m/sec;
R	is the radius of a gas bubble, m;
Δt	is the temperature drop, °C;
r	is the heat of evaporation, J/kg;
n	is the stoichiometric ratio, kg_{H_2}/kg_{gas};
g	is the acceleration of gravity, m/sec^2;
$\mathrm{Pr}_R = \nu/D_R$	is the Prandtl diffusion number for hydrogen ions;
$\mathrm{Pr}_G = \nu/D_G$	is the Prandtl diffusion number for gas;
Nu	is the Nusselt number;
Re	is the Reynolds number;
a	is the thermal diffusivity, m^2/sec;
σ	is the surface tension, N/m;
ν	is the kinematic viscosity, m^2/sec;
α	is the heat transfer coefficient, $W/m^2 \cdot °C$;
ρ'	is the density of liquid, kg/m^3;
ρ''	is the density of vapor, kg/m^3;
ρ_G	is the density of gas, kg/m^3;
τ	is the time.

Subscripts

G	denotes gas;
R	denotes reagent;
S	denotes surface.

Superscripts

' denotes liquid;
" denotes vapor.

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PRESSURE AND VELOCITY DISTRIBUTIONS IN A LAMINAR DISCHARGE OF FLUID FROM A PERFORATED PIPE

R. S. Kuznetskii

UDC 532.5

For a turbulent flow in a pipe with a sealed end we have, when the discharge through uniformly spaced perforations along the pipe is laminar ($\mathrm{Re} < \mathrm{Re}_*$ and $d\sqrt{p} \lesssim A \equiv \nu\sqrt{c\mathrm{Re}_*\rho/2}$)

$$\left(\frac{p}{\rho}+\alpha v^2\right)' = -\frac{\lambda}{2}v^2, \quad -\frac{L}{\sigma}v' = w = \frac{2}{c}\frac{d}{\mu}p; \; p(0) = p_0, \; v(L) = 0. \tag{1}$$

Here $\alpha = \varkappa/4$ is the Coriolis coefficient, σ is the ratio of total perforation area to inside pipe section area; ρ and $\nu = \mu/\rho$ are the density and the kinematic viscosity of the fluid, respectively; $l = Lx$ $(0 \le l \le L,$ $0 \le x \le 1)$ is the pipe length coordinate (subscripts 0 and 1 refer to $x = 0$ and $x = 1$, respectively); $p(l) = p_0h(x)$ and $v(l) = u(x)\sqrt{2p_0/\rho}$ are the mean-over-the-section excess pressure and the axial fluid velocity in the pipe, respectively; $\lambda = \psi/L$ is the referred pipe resistance coefficient; $w(l)$ is the discharge velocity through the perforations; d is the diameter of each perforation; $\mathrm{Re} = dw/\nu$ is the Reynolds number referred to the perforation diameter ($\mathrm{Re}_* \lesssim 10$ is critical); $\mathrm{Re}_0 = cr = (\sigma d/\nu)\sqrt{2p_0/\rho}$; $c = \text{const} \simeq 25.2$; and the values of A for various liquids and gases are tabulated.

In dimensionless form Eqs. (1) become

$$h' + u(\varkappa rh - \psi u), \quad u' = -rh; \; h(0) = 1, \; u(1) = 0, \tag{2}$$

and in this way define the criterial process numbers Re_0 and ψ. System (2) can be integrated in quadratures only in the extreme cases of very small or very large values of the pipe resistance coefficient; in the general case it can only be integrated numerically on a computer. The results of such a computer-aided numerical integration are shown in Fig. 1 for a typical value $\mathrm{Re}_0/\psi = 5$.

It follows from an analysis of system (2) that u(x) decreases everywhere monotonically while function h(x) has a maximum (h_1) at $x = 1$, increases monotonically when $u_0 < \varkappa r/\psi$ (i.e., at Re_0 smaller than some Re_0^*), and also has a minimum when $u_0 > \varkappa r/\psi$ (at x corresponding to $u/h = \varkappa r/\psi$).

The function $h_1(\mathrm{Re}_0)$ at $\mathrm{Re}_0/\psi = \text{const}$ has a minimum equal to unity at $\mathrm{Re}_0 = 0$ and a maximum at $\mathrm{Re} = \mathrm{Re}_0^*$, this maximum increasing with higher Re_0/ψ values. Function $u_0(\mathrm{Re}_0)$ is zero at $\mathrm{Re}_0 = 0$ and then increases asymptotically approaching a limit which becomes higher with higher Re_0/ψ values.

When $\mathrm{Re}_0 < \mathrm{Re}_0^*$, the values of the monotonically increasing function h(x) increase as Re_0 increases and ψ decreases. When $\mathrm{Re}_0 \ge \mathrm{Re}_0^*$, h(x) has a minimum which decreases (together with all values of h(x))

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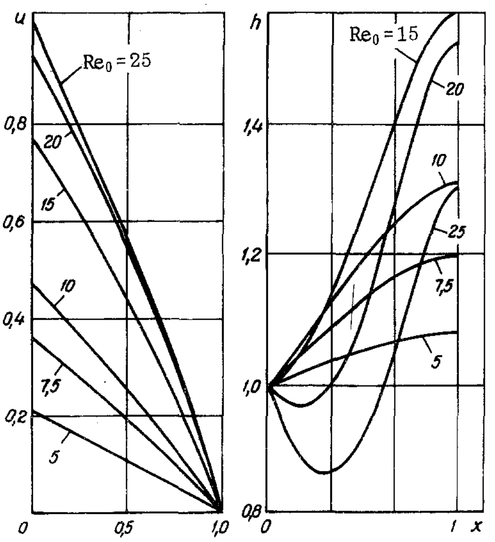

Fig. 1. Dimensionless excess pressure h and axial velocity u as functions of the dimensionless pipe length coordinate x, at various values of the Reynolds number Re_0 ($Re_0/\psi = 5$).

and shifts toward the right as Re_0 and ψ increase; moreover, the maximum h_1 becomes less peaked. The values of function u(x) increase as ψ decreases (more rapidly at higher Re_0 values) and essentially increase as Re_0 increases.

TEMPERATURE FIELDS IN MULTILAYER SEMITRANSLUCENT COATINGS HEATED BY RADIATION PULSES

V. V. Frolov

UDC 532.526.011:536.24

The transient one-dimensional heat conduction problem is analyzed for the case of a multilayer system, taking into account distributed absorption of heat from an external radiant source. The absorption process is assumed linear. Reflections and concentrated absorption at the interlayer boundaries are also taken into account. As an example, the temperature distribution problem is solved numerically for a two-layer protective coating. The external heat load on the coating is represented in terms of a triangular pulse of a given total energy content, and the intrinsic radiation from the surface of the outer layer is also accounted for.

The problem is solved numerically for pulses 60, 20, and 10 sec wide. The total energy in a pulse corresponds to actual intensities of radiant thermal fluxes passing into the atmosphere, assuming the passage occurs within about 10 sec.

The numerical solution to the heat conduction problem is obtained by the implicit scheme; the boundary-value equations at each step are solved by combining a reduction to the Cauchy problem with the sweep method. Moreover, the fundamental system of solutions is presented in the form of piecewise-continuous functions. The quantity of heat absorbed, as a function of time, is plotted on curves for various values of the absorptivity and for various pulse widths. Furthermore, instantaneous temperature distributions in the layers of semitranslucent and fully opaque systems at various instants of time are shown too.

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The results obtained here indicate that, disregarding the bulk absorption of radiant energy by the system, the errors in the calculated temperature distribution or quantity of heat can be quite appreciable even at high values of the absorptivity, and that this inaccuracy becomes greater with shorter pulse widths.